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# Interactive Writing in Mathematics Class: Getting Started

**M**any teachers are recognizing the advantages that student writing may provide in an academic mathematics environment—developing students' understanding of ideas (Pugalee 1997), enabling teachers to identify incomplete conceptions (Chapman 1996), and contributing to a more interactive relationship between students and teacher (Dougherty 1996). However, writing to learn mathematics is not something that most teachers experienced when they were students. They seek answers to many questions so that their first efforts with student writing can be effective:

- Why do students write in mathematics class?
- What prompts should I use?
- What outcomes can I address through interactive writing?
- How should I respond?
- What about the practical questions: When, how often, and for how long?

We have found that these questions interrelate—the answer to any one question depends on the answers to the others. This article uses examples of a student's writing in a tenth-grade academic mathematics program to help us respond to each question. At the same time, we hope to demonstrate the benefits provided by an *interactive* approach to writing for learning in the mathematics classroom.

Interactive writing is not complicated. The teacher initiates the process with a prompt that suggests a topic of mutual interest. Within the bounds of the prompt, the students each choose what to share with the teacher in their responses. After reading each student's response, the teacher writes a brief reply. With interactive writing, the teacher and the students get to know each other's views on matters relevant to learning mathematics. We sometimes design prompts to invite students to express mathematical concepts in words. At other times, the prompts invite students to address other aspects of the challenge of learning mathematics. As the examples in this article show, an interactive writing program is most valuable when, over time, it deals with a range of matters regarding the chal-

lenges of learning mathematics. When the teacher and the students develop more interest in each others' views, the writing becomes an interactive aspect of the teacher-student relationship, rather than just a task for students to complete.

The first example is taken from a student's response, near the midpoint of an academic tenth-grade mathematics course. It is the seventh exchange of writing of the year and was written after the fourth unit test of the year was returned. Karim is describing how he studied.

I sat at the table just after I had fixed the stereo. I turned it up until my ears began to ring, then I opened my binder and looked at the review. Just then my brother Joseph and his bud Tom came in and began reading *Truckin* magazines. I joined in. Finally after I decided it was time to study, I closed my book and read the notes downstairs and I chatted with my friends online. That's why I only got 69%.—Karim

## WHY DO STUDENTS WRITE IN MATHEMATICS CLASS?

As his writing suggests, Karim's marks were not as good as he hoped, and he was not doing very well with his study habits. What the writing does not state is that Karim was not doing his daily mathematics with any more rigor than he was bringing to his studying. Karim was not adjusting well to the challenges of a mathematics course designed for the academic elite. The previous year, Karim had achieved marks of 90 percent and higher with little effort in a mathematics program whose curriculum was designed for all ninth-grade students. Karim did not need to have good study habits then, and he had

*The writing becomes an interactive aspect of the teacher-student relationship*

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assumed that he would not need to develop them. After his second test, for example, he wrote the following in his third written response of the semester.

I didn't study for the math test. I have never studied for a math test before, and it hasn't had any negative impact on my mark.

That comment from early in the semester shows where Karim was coming from, before his test scores dropped below his expectations. Even so, we cannot be sure what Karim meant when his later writing connected the lower test score with his poor studying. When Karim wrote, "That's why I only got 69%," was he justifying his low score because of interruptions, or was he opening himself to the teacher's input on his approach to studying? A teacher who writes a reply in interactive writing must consider the intentions that are behind the student's words.

We find that, in general, students write with as many as four reasons in mind. When the teacher provides a specific prompt and collects students' writing, whether or not other reasons exist as well, students are writing because the teacher told them to do so—*writing to respond*. This reason is generally the dominant one when students begin writing in our classes. Generally they want to cooperate with their teachers, and often they do not have any idea of any other reasons that they might have for a process that they do not yet understand.

As they participate further in that process, students usually find themselves with fewer teacher-centered reasons for what and how they write. Sometimes they just want to let the reader, that is, the teacher, know something. We call this intention *writing to report*. It is similar to writing to respond, but the writer is choosing what to write, as well as choosing how much to write and how to write it. The students sometimes write while they are considering a concept about which they are uncertain. In *writing to reflect*, something that was at best an implicit nonverbal idea becomes the object of conscious consideration. Finally, students may intend that their writing affect the student-with-teacher relationship. We call this last intention *writing to relate*. Writers with this intention are aware of their audience, that is, the teacher, and the relationship that exists between them; these students in particular look forward to the teacher's written reply. We can reasonably attribute any of these four intentions to Karim. A more insightful interpretation should wait until we are able to discuss his writing over a period of time.

## WHAT PROMPTS SHOULD I USE?

The two prompts that triggered Karim's responses were quite simple. Both prompts asked about studying. After the second test, the teacher asked,

"How did you study for the test? Did studying affect your mark?" Two tests later, the prompt that led to Karim's description of his unsuccessful attempt to study was, "How did your studying for this test compare with your studying for the earlier test, as you described in written response 3?"

In both situations, the teacher believed that students might benefit from considering the question and writing a personal answer. Also, the teacher believed that she was likely to get more than predictable or generic responses—in fact, she thought that the answers might help her understand the writers as individual students. The question is therefore relatively authentic. It was not a question with a single right answer that the teacher already knew. The answers could give information that the writers would want the teacher to have, information that the teacher could incorporate into her decision making, and information that would furnish a starting point for the teacher's reply.

Prompts need to point to a place where all students can begin. Students find that putting into words something that they are doing or something that they know is easier than discussing something that they think or believe. The students add their thoughts and beliefs to their descriptions of actions or knowledge when their intentions have developed beyond simply writing to respond. Because each student is capable of deciding whether to report, reflect, or relate, according to his or her personal goals with the question, we do not need to make our prompts too directive. Also, pointing toward a specific topic for discussion is helpful—in the preceding situations, it was a single test score and the student's preparation processes. Each student can choose the aspect of his or her preparation that he or she writes about, and the student can choose the approach to take to interpret the test score in relation to his or her studying. In Karim's responses, his concern about his approach to studying was taking form, and his commitment to action was developing. The prompts gave him a chance to think about his test scores and his studying methods but allowed him to choose how to think about the topics and what to share.

In Karim's class, the next writing prompt was similar. It came two weeks and one unit test later, on the day that the students found out their midyear mark to date. It asked them, "Look back at the first time you wrote to me, where you stated your goals for the course and what you planned to do to achieve them. Interpret your midterm mark in comparison with those goals and plans." The following is Karim's response:

Well I reached for the stars and I made it. I passed with a 79%. Maybe for the next term I'll try to break 80%. (I'm really pushing myself.) I

***The teacher thought that the answers might help her understand the writers as individual students***

***“I studied more, but I got worse. What is my problem?”***

think if I half pay attention I could go AP for all three years.

For the last test I studied more but I got worse. What is my problem? I come from a crazy house and I have a short attention span. More studying for me is like maybe a whole ten minutes of almost full concentration.—Karim

### WHAT OUTCOMES CAN I ADDRESS THROUGH INTERACTIVE WRITING?

The preceding examples show that students' writing can be a way for teachers to support students' development of study skills. Students do need to think about and express what they are trying to do as learners and how they are going about it. For instance, responses to questions about studying by a range of students have suggested to us that different students need to study differently. Students who have a general grasp of all concepts may benefit most from a surveying process, in which they attend to terminology and key examples the night before a test. In contrast, students whose understanding is incomplete may need to begin studying much earlier so that they have time to get further instruction and learn the details in their areas of difficulty. A later step for these students is to build their own confidence with survey-style studying just before the test. In our classrooms, students' writing has helped them grapple with these ideas and has helped us recognize this kind of distinction. Our responses to our students give us the chance to advise students on their approaches to test preparation.

However, over time, students' writing has to address a variety of purposes so that it can be useful to every student who must participate. For instance, students can be directed to write mathematically. In such a situation, students are being directed to respond for the purpose of reporting what they know. “A great deal can be learned from students when they are asked to write about their understanding of mathematics” (Miller 1991, p. 518). At the same time, students can develop precision in their use of mathematical vocabulary and enrich the conceptual understandings of the topic of their writing. “Teachers should help students become more precise in written mathematics” (NCTM 2000, p. 351).

The following is Karim's response when his class was asked to describe two kinds of factoring that they remembered from the previous year. Karim's response, along with those of his classmates, helped the teacher answer an important question: how much intervention would such students as Karim need before the teacher could build more advanced content onto their knowledge of basic factoring?

Perfect squares  $(x + 5)^2 = (x + 5)(x + 5)$

1. Using the variable  $(x)$  in the first bracket, mul-

tiple by the variable and the constant in the second bracket.

- Using the constant in the first bracket, multiply it by the variable and constant (5) in the second bracket.
- Add like terms.

Difference of squares  $(x + 3)(x - 3)$   
 $x^2 - 3x + 3x - 9$   
 $x^2 - 9.$

Karim benefited from having shown his teacher what he did and did not understand. The teacher-reader can see that Karim had accurate recall of specific processes but that he would benefit from help in distinguishing factoring from expanding before engaging with the unit's new material.

The topics for interactive writing shown in this article include the students' understanding of particular mathematical ideas and the challenges that students face when they study for mathematics tests. When teachers are more experienced with interactive writing, they may want to use interactive writing to address other challenges. These challenges may include mathematics anxiety, students' identity in mathematics class, and students' sense of what mathematics is (Borasi 1990). However, teachers who are starting to use interactive writing may want to wait until they and their students are more experienced with writing in mathematics before tackling such issues. If a teacher starts interactive writing with writing about what students do to succeed with mathematics and includes writing about specific aspects of mathematics content (Masingila and Prus-Wisniowska 1996), the teacher will have the benefit of examples of the students' thinking before choosing topics that involve more vulnerability.

### HOW SHOULD I RESPOND?

Responding to student writing is intimidating to teachers who want to begin to use writing. First, knowing what to say is difficult. A second reason is that the prospect of finding time to write individual responses to even one class of students is daunting. The second concern is easier to deal with. We do not often ask students to write. And we do not take longer to respond to a class set of students' writing than we would take to mark a set of homework questions. We must trust that the students will recognize that the sentences that we write are our first thoughts about how to respond and that we must write just a few sentences to each one. However, carefully choosing those sentences is important.

We attempt to address the individuality in each student's response, and then we try to suggest action regarding any goals that the student mentions. We believe that by writing meaningful replies rather than generic platitudes, we model for the students

how to write with purpose. Because we think of interactive writing as just one facet of the relationship that we develop with each student, we do not demand of ourselves that we be accurate, perfect, or complete each time that we write. We respect the limits of our capabilities by focusing our responses on what we want to influence. We write as the students' mathematics teacher, not their friend—advising, correcting, judging, and encouraging.

The following is the teacher's reply to Karim's early statement that he did not need to study. She is respectful but practical, offering well-meant advice that includes a specific starting point, in case Karim is ready to consider taking action.

Karim, thanks for being honest about not studying. And while this might be acceptable for right now, I think that you may find that the quality of your learning and your marks may suffer. For the next test, try to redo some of the tougher exercise questions and review your notes.—Mrs. McFeetors

By the second entry about studying, the teacher was more confident in her knowledge about Karim. She chose to be blunt and to give advice.

Karim, you need to start sitting down and doing more work for this course. Start by finishing your homework every day and spending about 1–1.5 hours to study for the next test.—Mrs. McFeetors

For the next entry, the teacher chose to be encouraging, responding to the upbeat tone of Karim's statements about his midterm mark. The teacher's response is longer than average and respects Karim's specific call for advice on a complex matter by offering advice that interacts with the details that Karim wrote in his entry.

Karim, you're doing great! If you put in the time and effort, I think that you can do the AP all through high school! If you have trouble concentrating, choose one small topic (like finding the LCD) and spend 10 minutes on it. Take a break, and then come back and spend another 10 minutes on a different small topic. Try spending 30–40 minutes a day. If it gets crazy at home, try spending some of this time at school or working with a friend at their place or a library. Hope this helps—keep up the good work.—Mrs. McFeetors

The teacher's response to the writing about factoring was different. She had asked the students to express what they knew about a mathematics topic. The teacher chose to give this entry a holistic score and treat it as a quiz.

Karim 7/10.

Factoring is going the opposite way. This is expanding.

### WHAT ABOUT THE PRACTICAL QUESTIONS: WHEN, HOW OFTEN, AND FOR HOW LONG?

Keeping the educational intentions of interactive writing in mind can help mathematics teachers with their practical decisions. For instance, how long should students' responses be? Students always ask for guidelines on the length that we expect, and we usually suggest that one or two well-written paragraphs can provide students with a chance to give a general response and some supporting details. When writing in class, ten minutes is enough time to allow all students to be thoughtful about the prompt.

The timing of interactive writing can vary within the class period. For the examples shared previously about test scores, students could be asked to respond at the beginning of the period, as soon as they receive their mark. Or they could write in the last ten minutes, after they have had a chance to see, discuss, and reflect on what they did to earn that mark. When we ask students to write about mathematical concepts, we tend to have them write at the beginning of class. For instance, to assess students' understanding from a previous day's lesson, Karim and his classmates were asked the following: "Say in words how to determine a line's slope from its graph, without using the line's equation."

Karim wrote the following:

To find the slope of a line you divide the rise by the run. However high it is from the first point is the rise and however far it is from the bottom point to the top along the bottom is the run. Now divide rise by run.

Karim's teacher was encouraged that Karim had translated the technical procedures of the previous day into a conceptual blend of formal and natural language (Bagley and Gallenberger 1992).

Another concern is how often students should write and how often the teacher should provide full responses. For Karim's class, the students wrote ten entries (asterisks indicate the entries discussed in this article):

1. The student's course goals and plans to achieve the goals
- \*2. What the student recalls of two factoring techniques
- \*3. How the student studied for test 2 and whether the mark reflects how he or she studied
- \*4. Description of slope

*We model  
for the  
students  
how to write  
with  
purpose*



**What they  
write is  
more likely  
to be worth  
reading**

5. In what ways the student takes notes during a mathematics lesson
6. Explanation describing how to simplify a radical expression
- \*7. How the student studied for test 4 in comparison with test 2
- \*8. Reflection on midsemester report-card mark
9. Connecting a function's graph with its meanings
10. Reflecting on the course as a whole

In a semester course, Karim's class wrote approximately once every other week. The teacher read all responses every time, but wrote extended responses only for those entries that dealt with students' learning processes or marks. To avoid disappointing students, we tell them before they write whether they can expect a written reply the next day.

### A PLACE TO BEGIN

Writing to learn can be a positive component of an academic mathematics program, but its effectiveness depends on whether the students and the teacher see how they might benefit from their efforts. The examples in this article show that students may develop a variety of reasons for writing in mathematics class—to *respond* to a teacher's request for evidence of mathematical understanding, to *report* what they want the teacher to know, to *reflect* on their own learning, and to *relate* to the teacher as a partner in their learning. Each of those intentions points toward a different benefit for particular students. Karim is a good example of a student who found that he needed to develop a richer approach to mathematics, and interactive writing was a chance for him to come to terms with that goal. With an interactive orientation to their writing, students are likely to understand that writing is a process of communication, not just evaluation.

The teacher also needs to feel rewarded for her or his efforts. We believe that the right prompt can

communicate that the teacher has a desire for the information that the student can give, as well as an interest in the student. We hope that the examples in this article show that the teacher's responses can provide students with a sense of audience for their writing and recognize the student as a particular person with something significant to say. As well as completing the communication cycle with each student, a teacher's reply can model the kind of thoughtfulness that the teacher would like to see in the students' writing. When students perceive the teacher as an authentic and interested audience, what they write is more likely to be worth reading. The teacher's role in the interactive writing process will seem worthwhile when the teacher feels informed by what the students write.

The examples presented in this article are from a teacher's first attempts to use writing in her mathematics classes. We hope that, with Karim's help, we have shown some rewards that are possible with interactive writing and illustrated some strategies that can make a teacher's first efforts rewarding.

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